Plasma Waves in Magnetized Plasmas

How they are used in magnetic fusion research

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Lecture Outline

- Introduction
- Basic Wave Equations
- Wave Dispersion Relation
- CMA Diagram
- Electron Cyclotron Waves
- Lower Hybrid Waves
- High Harmonic Fast Waves
- Discussions

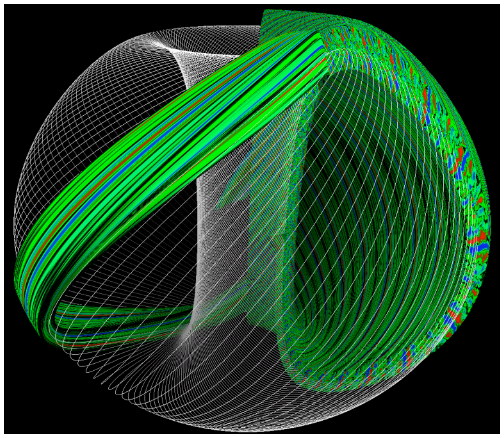
Why plasma waves?

- Plasma waves provide important means to transport **<u>energy and information</u>** in plasmas particularly across magnetic fields.
- Waves are used for heating, current drive, and plasma diagnostics in fusion research.
- In magnetized plasmas, there are many types of waves due to increased degrees of freedom.
- The magnetic field provides springiness and ion and electron cyclotron (gyro) motions, in addition to the plasma oscillations (John Finn Lecture)

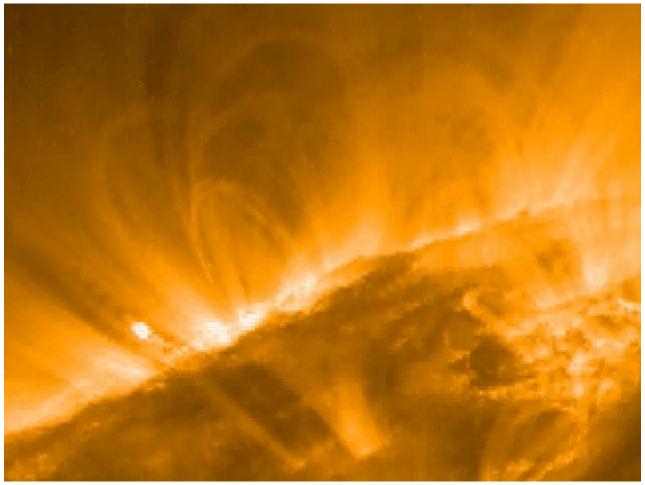
Ref: Waves in Plasmas by Thomas H. Stix, AIP 1992

Magnetized Plasma Waves

- In the fusion magnetic bottles, er need to transport energy across the magnetic fields to perform desired work (heating CD, diagnostics, etc.)
- In fusion plasmas, plasma waves propagates in very wide plasma parameter space.
  - Near vacuum to dense plasma
  - Near room temperature to 100's of millions of degrees
- In space, magnetic fields are present nearly everwhere.



Tokamak simulation

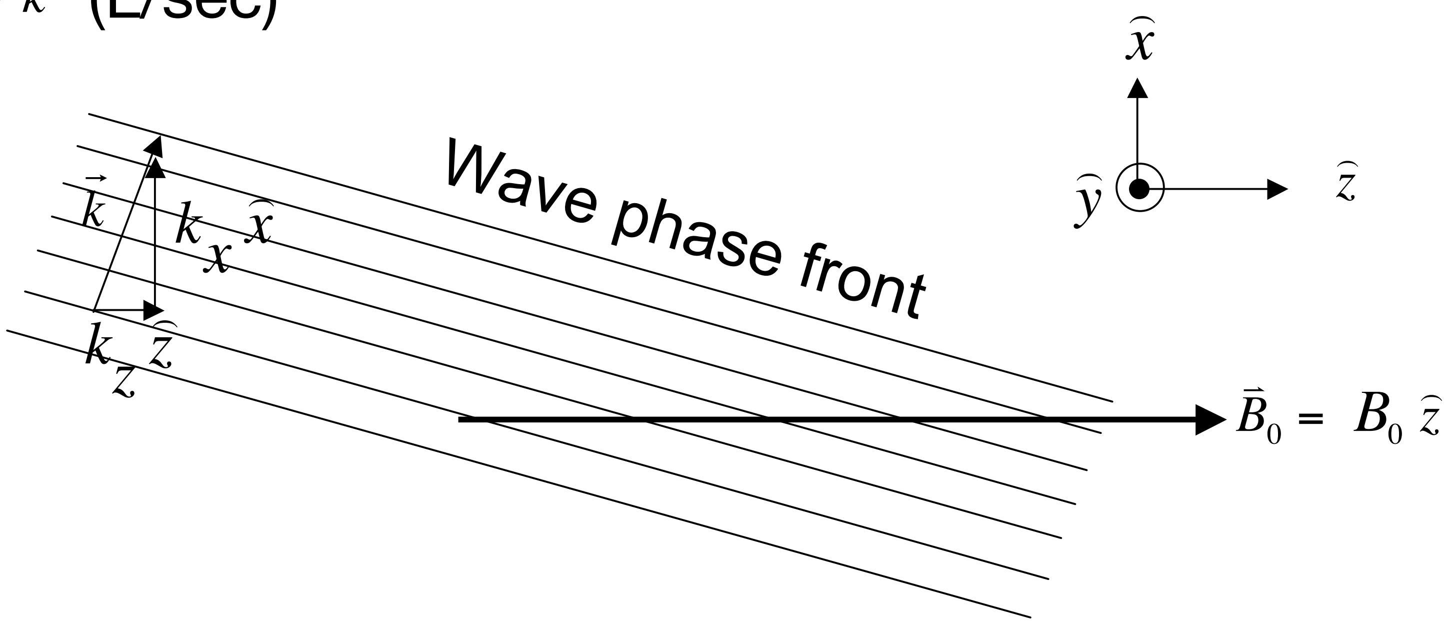


Trace Satellite Images

Cold Homogenous Plasma Waves

Basic Terminologies and Assumptions I

1. Assume ambient plasma and magnetic field $\vec{B}_0 = B_0\,\hat{z}$ are static in time $\tau >> 1/\omega$ and uniform in space $L_{Gradient} >> 1/|k|$
2. Assume a single frequency in radian $\omega = 2\pi f$ (1/sec) and a single wave number $\vec{k} = k_x\,\hat{x} + k_z\,\hat{z}$ (1/L)
3. Velocity of the wave phase front or phase velocity is $\omega / \vec{k}$ (L/sec)

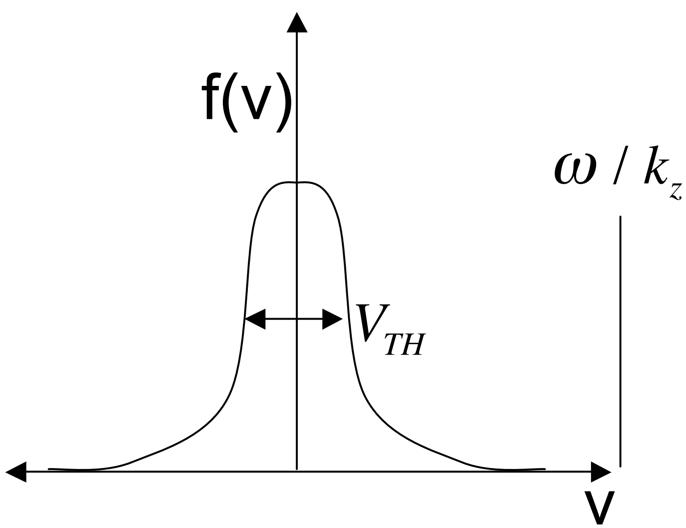


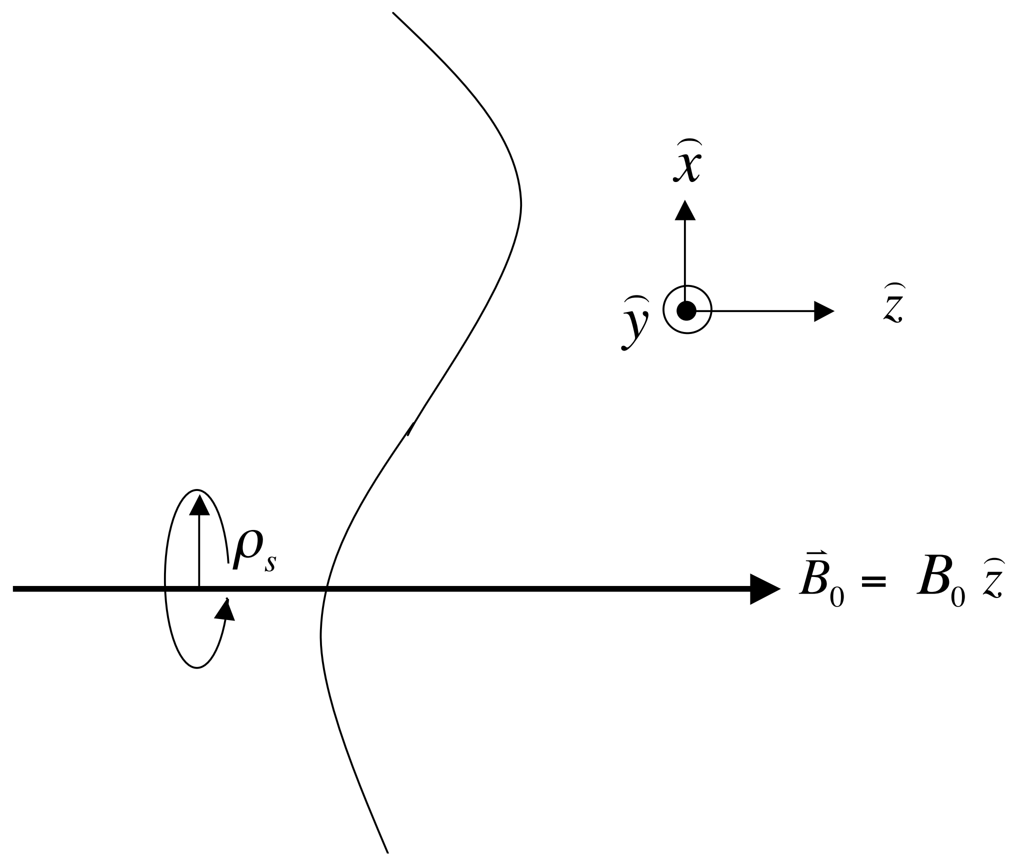
$$Note: \omega/\vec{k} = \omega\vec{k}/|k|^2 \neq \omega/k_x\,\hat{x} + \omega/k_z\,\hat{z}$$

Cold Homogenous Plasma Waves

Basic Terminologies and Assumptions II

4. Cold? $\omega / k_z >> V_{TH}$ (along the field line)
5. Cold? $k_x << 1/\rho_s$ (perpendicular to field line)
6. Cyclotron or gyro frequency in radian $\Omega_s = q_s B_0 / (m_s c)$
7. Plasma frequency in radian $\omega_{ps}^2 = 4\pi n_s q_s^2 / m_s$





Basic cold-homogenous plasma wave equations

From Maxwell equations:

$$\nabla\times\vec{B} = \frac{4\pi\,\vec{j}}{c} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t} = \frac{1}{c}\frac{\partial\vec{D}}{\partial t} = \frac{1}{c}\frac{\partial\,\overleftrightarrow{\varepsilon}\cdot\vec{E}}{\partial t} \qquad \nabla\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$$

Where plasma dielectric tensor $\overleftrightarrow{\varepsilon}(\omega,\vec{k}) = 1 + \sum_s \chi_s(\omega,\vec{k})$

Fourier analysis in time $\partial/\partial t \rightarrow -i\omega$ and space $\nabla \rightarrow i\vec{k}$ gives the homogeneous-plasma wave equation

$$\vec{k}\times(\vec{k}\times\vec{E}) + \frac{\omega^2}{c^2}\overleftrightarrow{\varepsilon}\cdot\vec{E} = 0$$

Therefore the general cold-plasma dispersion relation can be derived if we can obtain the plasma response to the wave electric field

.

$$\vec{j}_s = \overleftrightarrow{\sigma}_s\cdot\vec{E} = -\frac{i\omega}{4\pi}\overleftrightarrow{\chi}_s\cdot\vec{E} \qquad \vec{j} = \sum_s \vec{j}_s = \sum_s n_s\, q_s\, \vec{v}_s$$

Plasma response to wave electric field

The equation of motion for species "s" (e.g. electrons and ions) is

$$m_s \frac{d\vec{v}_s}{dt} = q_s(\vec{E} + \frac{\vec{v}_s}{c} \times \vec{B}) - \cancel{\nabla \cdot \Phi_s / n_s}$$

where the fluid stress (pressure) tensor term, can be set to zero for the cold plasma assumption. Also noting that $\Omega_s = q_s B_0 / (m_s c)$ which is positive for positive ions with the magnetic field pointing in the positive z direction (assumed here).

$$-i\omega\,\vec{v}_s = \frac{q_s}{m_s}\vec{E} + \vec{v}_s \times \vec{\Omega}_s$$

$$\omega_{ps}^2 = \frac{4\pi n_s q_s^2}{m_s}$$

In x-y coordinate, one can write

$$\begin{pmatrix} -i\omega & -\Omega_s \\ \Omega_s & -i\omega \end{pmatrix} \begin{pmatrix} j_x \\ j_y \end{pmatrix} = \frac{n_s q_s^2}{m_s} \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \frac{\omega_{ps}^2}{4\pi} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

Plasma Dielectric Tensor

Inverting the matrix, plasma response to electric field is

$$\begin{pmatrix} j_x \\ j_y \end{pmatrix} = -\frac{i\omega}{4\pi}\begin{pmatrix} \chi_{sxx} & \chi_{sxy} \\ \chi_{syx} & \chi_{syy} \end{pmatrix}\cdot\begin{pmatrix} E_x \\ E_y \end{pmatrix} = -\frac{i\omega}{4\pi}\frac{\omega_{ps}^2}{\Omega_s^2-\omega^2}\begin{pmatrix} 1 & i\Omega_s/\omega \\ -i\Omega_s/\omega & 1 \end{pmatrix}\cdot\begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

Generalizing to *x-y-z, one obtains*

$$\overleftrightarrow{\varepsilon}\cdot\vec{E} = \begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix}\cdot\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

where

$$S = 1-\sum_s\frac{\omega_{ps}^2}{\omega^2-\Omega_s^2}, \quad D = \sum_s\frac{\Omega_s}{\omega}\frac{\omega_{ps}^2}{\omega^2-\Omega_s^2}, \quad P = 1-\sum_s\frac{\omega_{ps}^2}{\omega^2}$$

Polarization drift | E x B drift | Inertial respose

Putting all together!

The combined Maxwell equation: $\vec{k}\times(\vec{k}\times\vec{E})+\frac{\omega^2}{c^2}\vec{\vec{\varepsilon}}\cdot\vec{E}=0$

The refractive index: $\vec{n}\equiv\vec{k}\,c/\omega=n\sin\theta\,\hat{x}+n\cos\theta\,\hat{z}$

Then the combined Maxwell equation becomes

$$\vec{n}\times(\vec{n}\times\vec{E})+\vec{\vec{\varepsilon}}\cdot\vec{E}=0 \qquad \vec{n}\times(\vec{n}\times\vec{E})=(\vec{n}\cdot\vec{E})\,\vec{n}-(\vec{n}\cdot\vec{n})\,\vec{E}$$

Then:

$$\vec{n}\times\vec{n}\times\vec{E}=\begin{pmatrix}-n_z^2 & 0 & n_x n_z\\ 0 & -n^2 & 0\\ n_x n_z & 0 & -n_x^2\end{pmatrix}\cdot\begin{pmatrix}E_x\\ E_y\\ E_z\end{pmatrix} \qquad \vec{\vec{\varepsilon}}\cdot\vec{E}=\begin{pmatrix}S & -iD & 0\\ iD & S & 0\\ 0 & 0 & P\end{pmatrix}\cdot\begin{pmatrix}E_x\\ E_y\\ E_z\end{pmatrix}$$

Then the cold magnetized homogeneous plasma wave equation becomes

$$\begin{pmatrix}S-n^2\cos^2\theta & -iD & n^2\sin\theta\cos\theta\\ iD & S-n^2 & 0\\ n^2\sin\theta\cos\theta & 0 & P-n^2\sin^2\theta\end{pmatrix}\begin{pmatrix}E_x\\ E_y\\ E_z\end{pmatrix}=0$$

$$S=1-\sum_s\frac{\omega_{ps}^2}{\omega^2-\Omega_s^2},$$

$$D=\sum_s\frac{\Omega_s}{\omega}\frac{\omega_{ps}^2}{\omega^2-\Omega_s^2},$$

$$P=1-\sum_s\frac{\omega_{ps}^2}{\omega^2}$$

Wave dispersion relations

Wave dispersion relations can be derived by taking the determinant of the wave equation matrix. Very conveniently, the resulting equation is quadratic in n^2!

$$A\, n^4 - B\, n^2 + C = 0$$

where

$$A = S \sin^2\theta + P \cos^2\theta$$

$$B = RL \sin^2\theta + PS\,(1 + \cos^2\theta)$$

$$C = PRL$$

The solution is: $$n^2 = \frac{B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$S = \frac{1}{2}(R + L)$$

$$R = 1 + \sum_s \chi_s^- = 1 - \sum_s \frac{\omega_{ps}^2}{\omega(\omega + \Omega_s)},$$

$$L = 1 + \sum_s \chi_s^+ = 1 - \sum_s \frac{\omega_{ps}^2}{\omega(\omega - \Omega_s)},$$

$$P = 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2}$$

- There are two wave solutions - "fast" and "slow"
- $n \to \infty$ or wave phase - resonance - absorption - electrostatic waves when $A = 0 \quad or \quad \tan^2\theta = -P/S$
- $n \to 0$ or wave phase - cutoff - reflection when $P = 0 \quad or \quad R = 0 \quad or \quad L = 0$

Clemmow-Mullaly-Allis Diagram

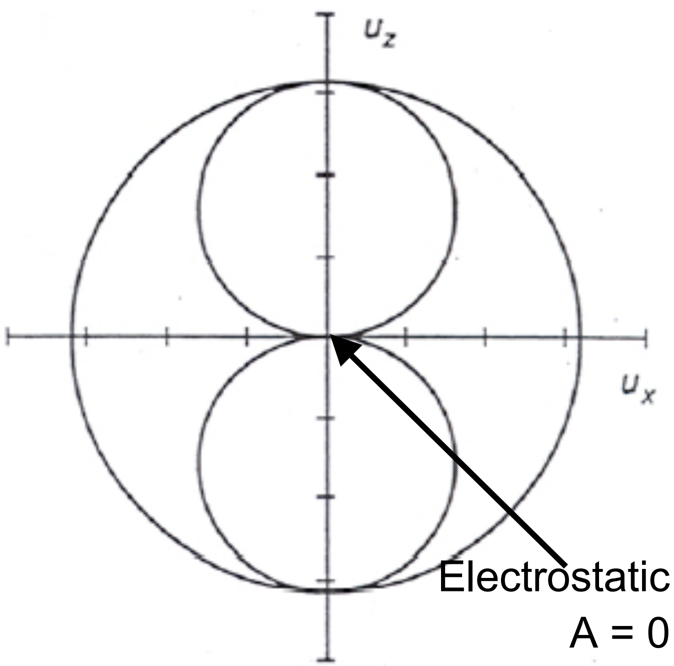
With $u = \omega / kc = 1/n$

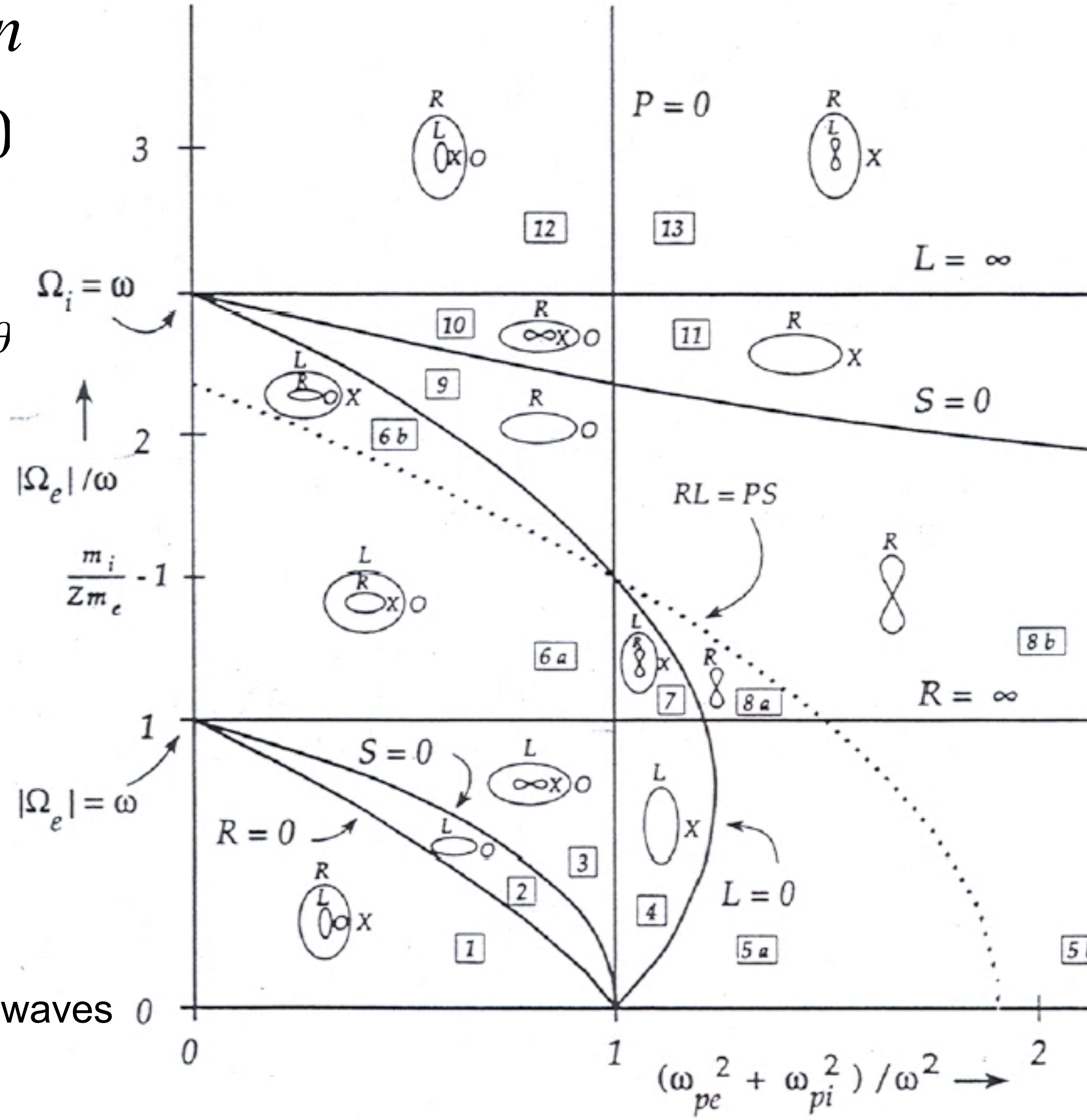
$$C\,u^4 - B\,u^2 + A = 0$$

$$A = S \sin^2\theta + P \cos^2\theta$$

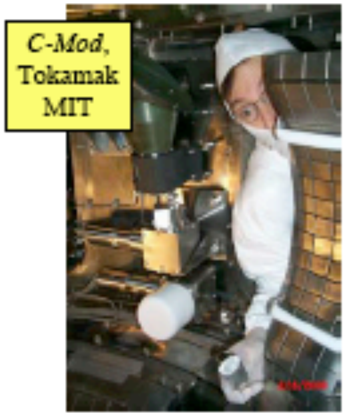
$$B = RL \sin^2\theta + PS\,(1 + \cos^2\theta$$

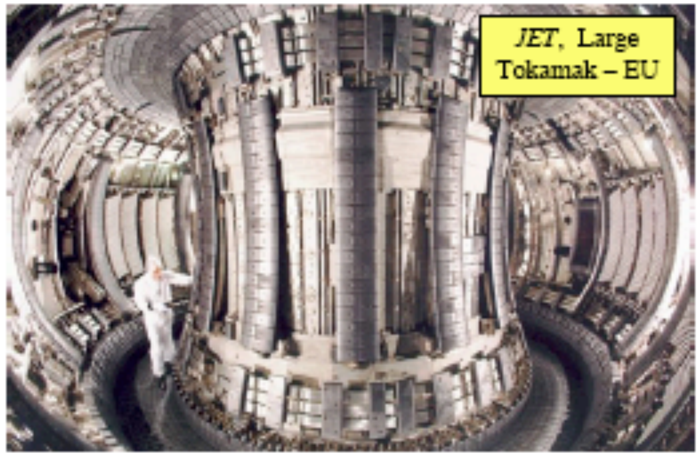
$$C = PRL$$

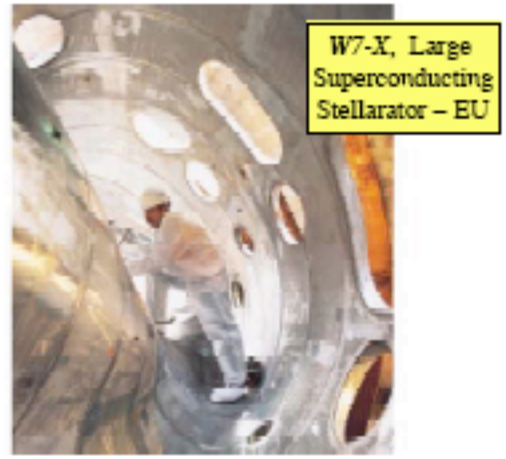


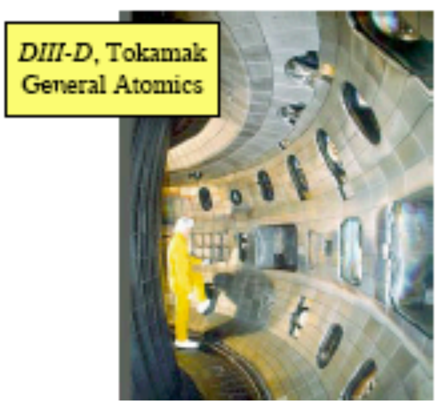


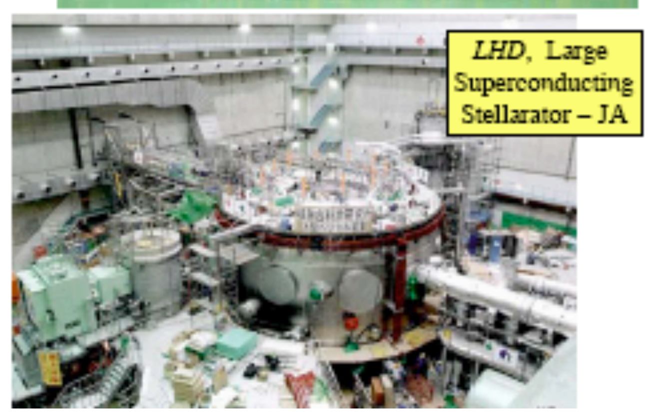
Magnetic Fusion Research is a Worldwide Activity, Optimizing the Plasma Configuration



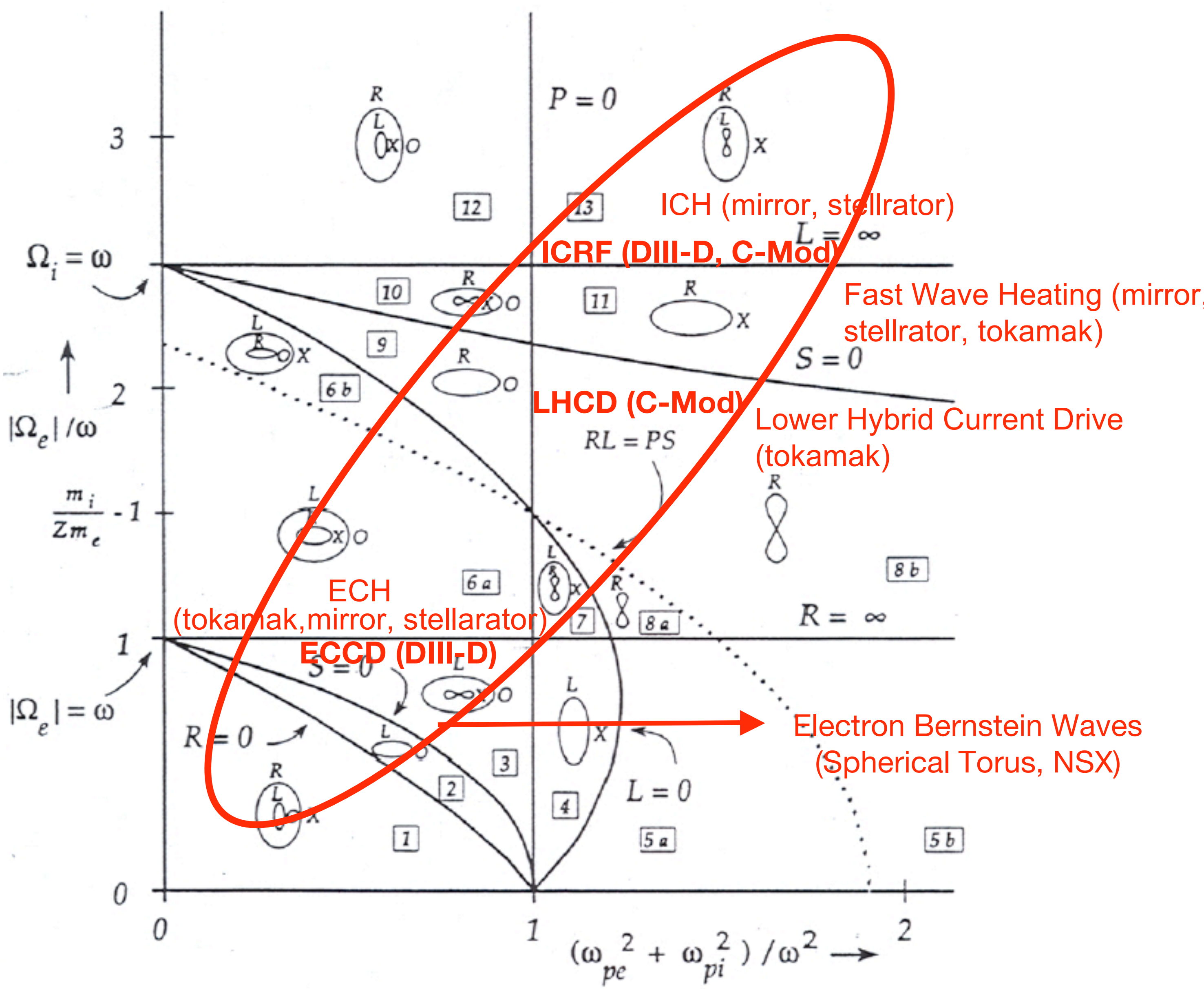




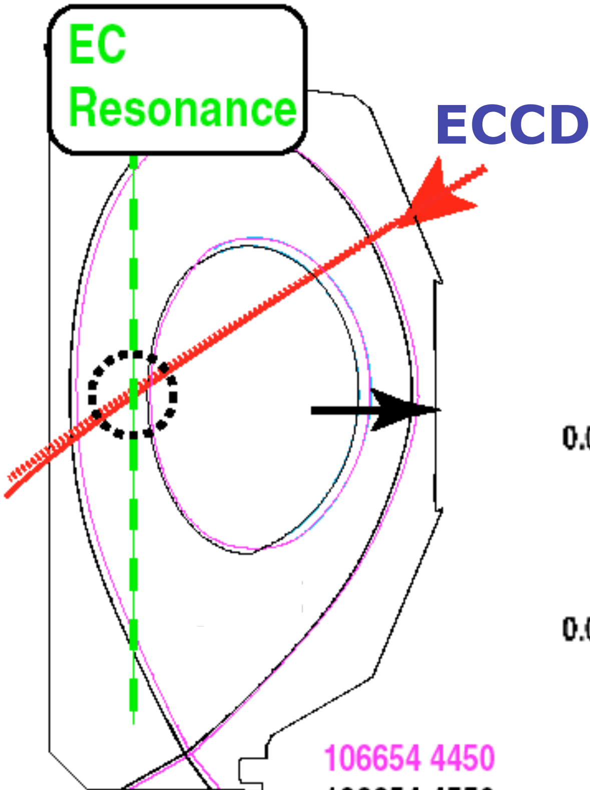


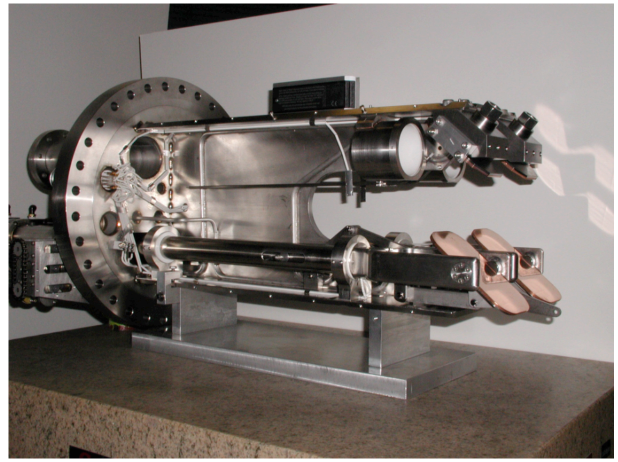


Fusion Plasma Applications

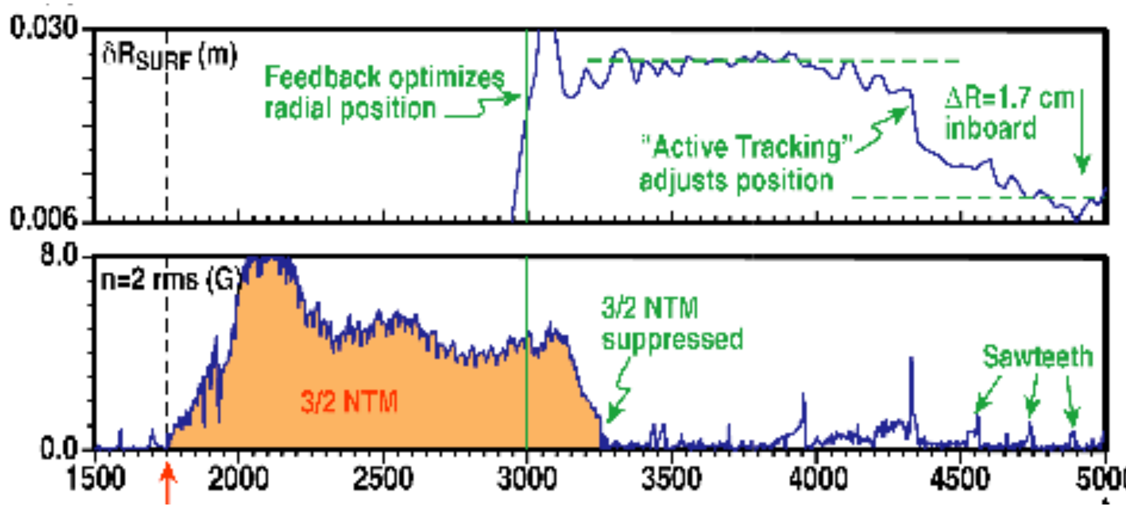


Application of RF Waves Stabilizes Neoclassical Tearing Modes





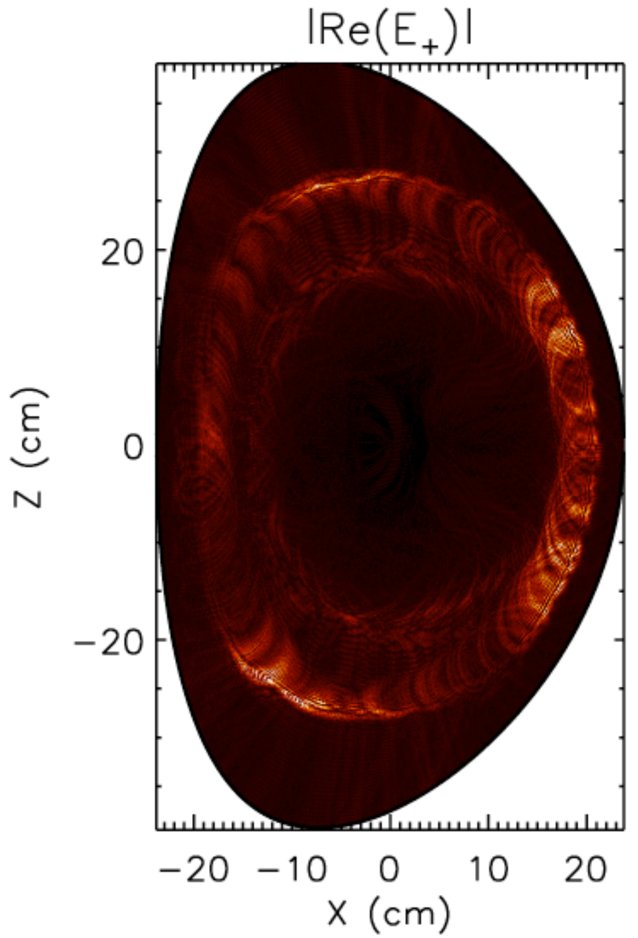
Steerable Electron Cyclotron Current Drive wave launcher built by PPPL in DIII-D collaboration.



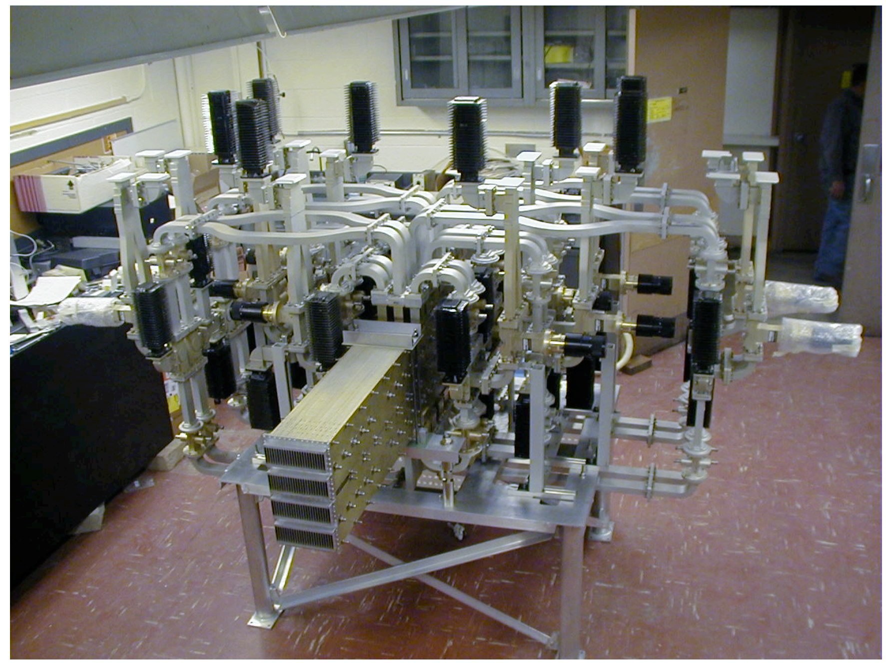
ITER will have ECCD for NTM control.
NCSX was designed to be stable to NTM modes.

GA

Lower Hybrid Current Drive (LHCD) will Enable Access to Advanced Tokamak Regime on C-Mod



TORIC code

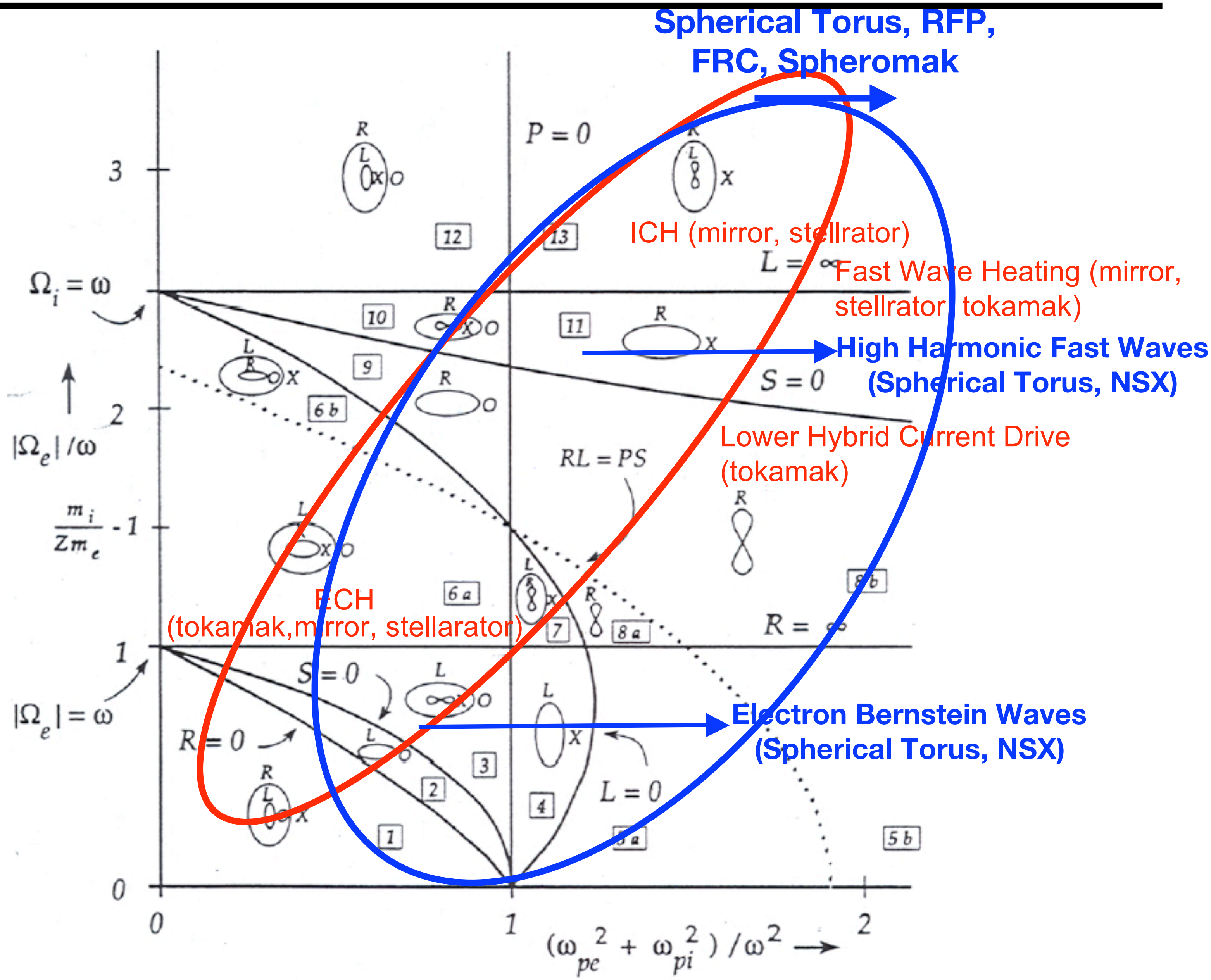


Collaboration on C-Mod has demonstrated efficient LH current drive

First ever full wave - Fokker Planck simulations for LHCD are being developed.

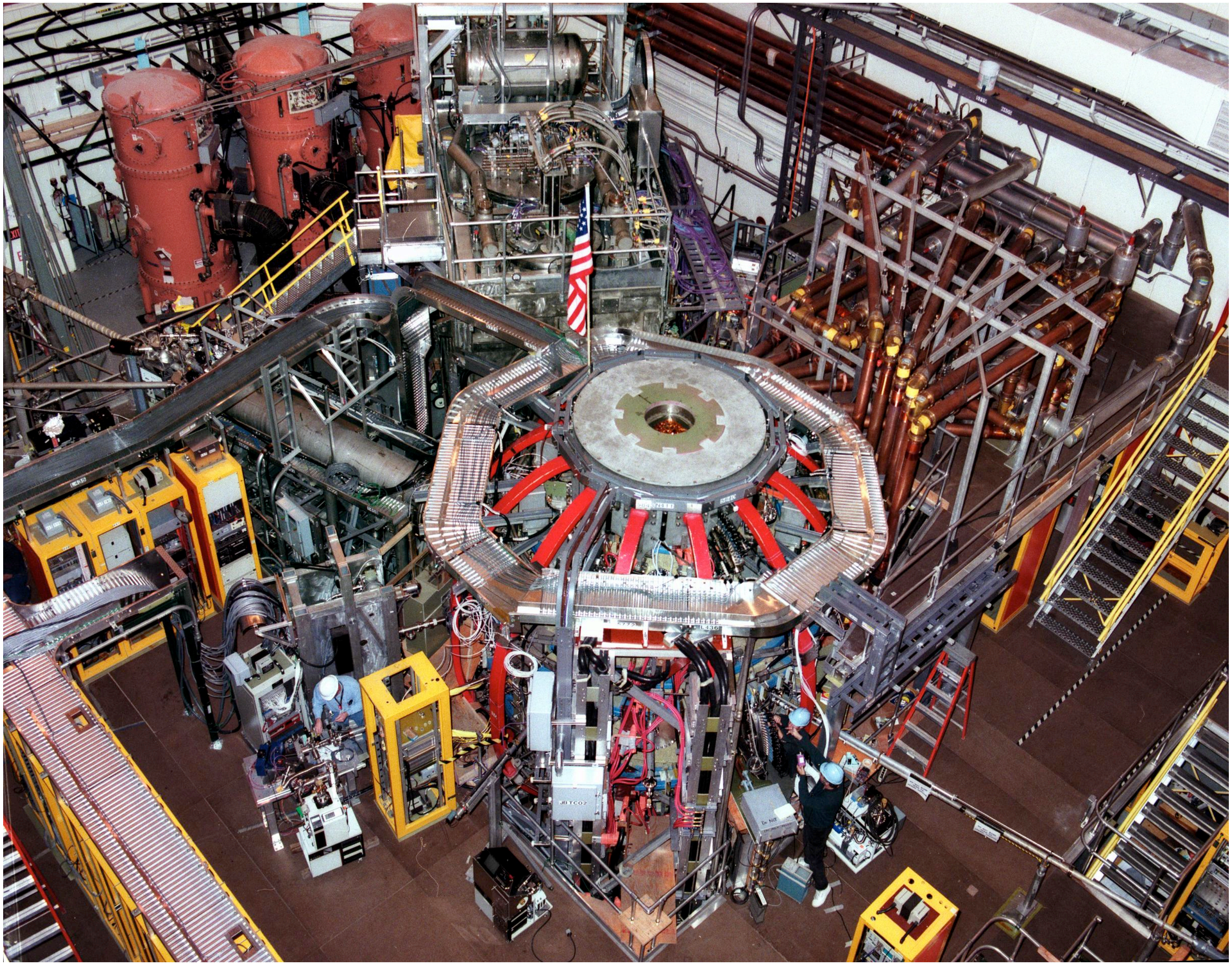
- Alpha particle damping on ITER will be addressed
- RF SciDAC collaboration between MIT, PPPL, ORNL, CompX, GA, Lodestar, and TechX

Heating "Overdense" Plasmas



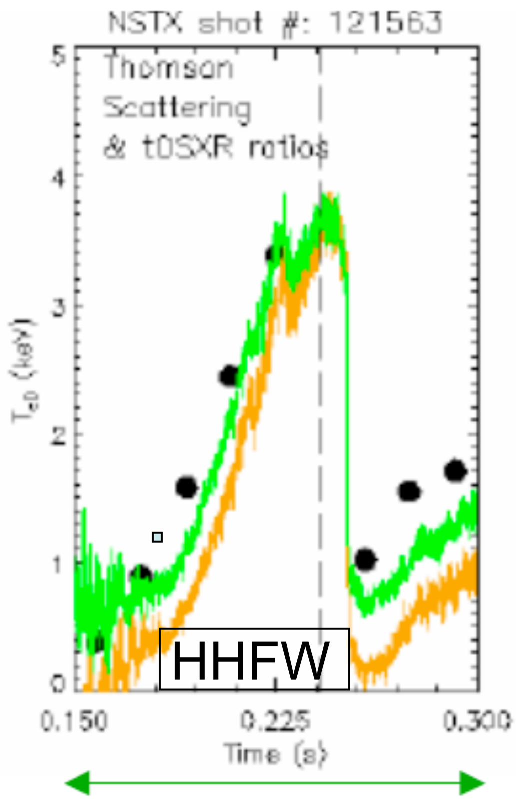
National Spherical Torus Experiment

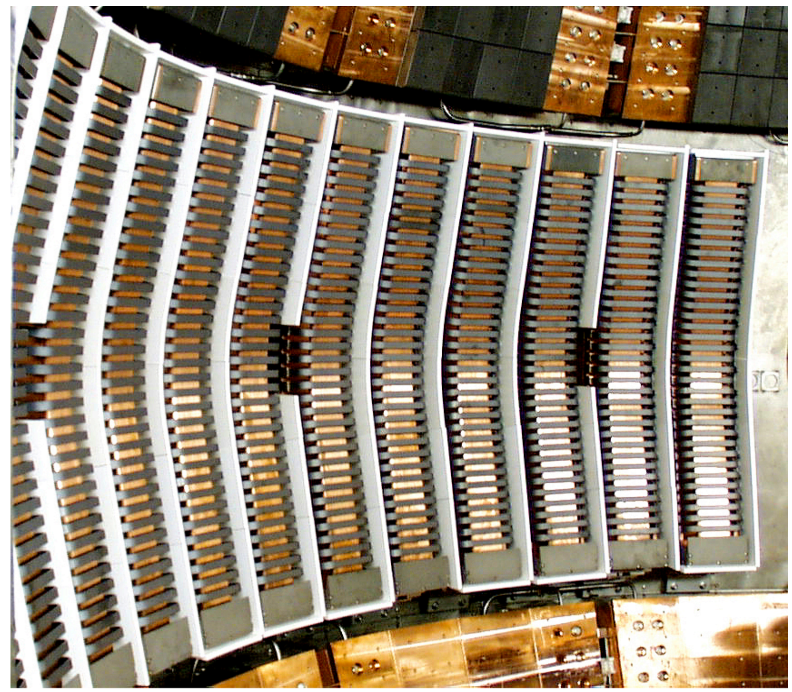
NSTX



HHFW in NSTX

- Bulk plasma heating to enhance bootstrap currents in advanced ST Operations
- Plasma start-up and current ramp-up
- Super-Alfvénic energetic particle physics
  - HHFW modification of NBI fast ion distribution function
  - TAE mode stablization





12 HHFW ANTENNA

12 antennas powered by 6 MW sources

ORNL, UCI, MIT, GA, CompX

Discussions

- **We derived a general dispersion relation of plasma waves in cold, uniform, stationary magnetized plasmas.**

- **In magnetized plasmas, there are many types of waves due to increased degrees of freedom.**

- **lasma waves are used for heating, current drive, and plasma diagnostics in fusion research.**

- **Plasma waves are also important in space physics.**

- **Plasma waves have other applications including semi-conductor fabrication, isotope separation, etc.**

Ref: Waves in Plasmas by Thomas H. Stix, AIP 1992